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Dynamical mechanisms of the temperature differences arising under uniform heat exchange conditions

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Abstract—Analytical study of dynamical mechanisms of the temperature differences between the elements of truss constructions under uniform heat transfer conditions is completed. The time-dependent temperature differences are associated with the different thermal time constants of the truss elements. In this study the truss elements are assumed to be the rods with low Biot numbers that allow one to consider them as thermally-thin bodies. The conductive lengths of the rods are supposed to be much smaller than the corresponding dimensions that allow one to neglect the conductive heat transfer. Analytical evaluations of the temperature differences are made for the following stepwise changing and oscillatory operating parameters: the ambient temperature; the heat flux density on the elements surface; and the convective heat transfer coefficient. The temperature differences are shown to be small for the two operating conditions limits. Firstly, for the case when all the time constants of the elements are much larger than the period of the operating parameters oscillation. Secondly, for the case when the time constants of all the elements are much smaller than the period of the oscillations.

1. INTRODUCTION

DEVELOPMENT of precise radio telescopes needs evaluations of the temperatures of the construction elements and the temperature differences between them. The temperature differences between the elements of truss constructions lead to considerable thermal deformations which affect the accuracy of the reflecting surface supported by the truss construction and the focusing efficiency of the antenna [1–3].

The elements of the truss constructions of radio telescopes operate under conditions determined by the following heat transfer factors: direct and earth-reflected solar radiation; infrared radiation; and convective heat transfer on the elements surface. It is evident, that the main temperature differences between the truss elements originate from the non-uniformity of the heat transfer conditions on their surfaces (due to the different space orientations of the elements). The steady-state temperature models of the radio telescope constructions are studied in the monograph [3] and shown not to be in full agreement with the experimental data.

In the practice of radio telescope development the temperature differences associated with this origin are suppressed by the following means. Firstly, by the use of coatings with high reflectance in the solar spectrum and high emissivity in the infrared spectrum. These coatings diminish the surface heat fluxes and, as a consequence, the absolute temperatures and the temperature differences in truss constructions [3]. The second way to decrease these temperature differences is the use of shells with insulation layers which enclose the truss construction of the antennas under the controlled ventilation inside them. This allows the protection of the truss construction from direct non-uniform radiative fluxes and, therefore, decreases the temperature differences between the elements of the truss.

But the above mentioned measures make it possible to diminish considerably only the steady state components of the temperature differences between the elements. With the development of the new generation of the millimeter radio telescopes it is necessary to consider more accurately their thermal regimes and to take into account the dynamical mechanisms of the temperature differences arising in truss constructions.

This paper is devoted to the analytical study of the temperature regimes of truss elements under dynamical heat transfer operating conditions, when the following parameters are considered as time-dependent: the ambient temperature; the heat fluxes on the truss surface associated with the radiation fluxes; and the convective heat transfer coefficient.

2. FORMULATION OF THE PROBLEM

The truss constructions have many rod-like elements which differ in geometrical and thermophysical properties. The elements are assumed to be in the airflow of forced or natural ventilation. The intensities of the convective heat transfer are assumed to be known in the form of heat transfer coefficients.

The heat transfer parameters on the surface of the truss are assumed to be the same for all elements. This

NOMENCLATURE

Bi_i Biot number, dimensionless
h_i convective heat transfer coefficient [W m^{-2} K^{-1}]
Δh amplitude of the convective heat transfer coefficient [W m^{-2} K^{-1}]
$\bar{h}$ harmonic mean of the convective heat exchange coefficient [W m^{-2} K^{-1}]
$\Delta h^* = \Delta h/\bar{h}$ dimensionless amplitude of the convective heat transfer coefficient
$h^* = h/h(0)$ dimensionless convective heat transfer coefficient
k_i heat conductance coefficient [W m^{-2} K^{-1}]
L_i length of the element [m]
Lc_i conductive length [m]
N total number of the elements
P_i heat transfer perimeter of the element [m]
q_s surface heat flux density [W m^{-2}]
Δq_s amplitude of surface heat flux density [W m^{-2}]
$\bar{q}_s$ harmonic mean of q_s [W m^{-2}]
S_i cross-sectional area of the truss element [m^2]
T_i temperature of the element [K]
T_a ambient temperature [K]
ΔT_{ij} temperature difference between the elements [K]
ΔT^*_{ij} amplitude of ΔT_{ij} [K]
T^*_s additional temperature of the element [K]
δT allowable temperature difference between the elements [K]
$\max\{\Delta T_{ij}\}$ maximal value of ΔT_{ij} [K]
ΔT_a amplitude of the ambient temperature [K]
$\bar{T}_a$ harmonic mean of T_a [K]
t time variable [s]
t_i thermal time constant of the element [s]
t_p oscillation period [s]
t_{max} time corresponding to $\max\{\Delta T_{ij}\}$ [s]
Δt_h characteristic time of convective heat transfer change
Δt_T characteristic time of ambient temperature change [s]
Δt_q characteristic time of heat flux density change [s].

Greek symbols
δ_i reduced diameter of the element [m]
ω frequency of oscillation of the heat transfer operating parameters [s^{-1}]
ω_i dimensionless frequency of oscillation of the element temperature.

Subscripts
i, j elements indices.

assumption is made to focus consideration on the dynamical components of the arising temperature differences.

The elements of the truss ($i = 1, \ldots, N$) are assumed to be long rods with the low Biot numbers:

$$Bi_i = \frac{h_i \delta_i}{k_i} \ll 1, \quad i = 1, \ldots, N \tag{1}$$

where $\delta_i = S_i/P_i$ are the reduced diameters of the elements given by the ratios of the rods cross-sectional areas S_i to the heat transfer perimeters P_i, h_i are the heat transfer coefficients and k_i are the heat conductance coefficients of the elements.

This condition allows us to consider the cross-section temperature distribution of the elements as the uniform, to neglect the cross-sectional derivatives in the conductivity equation and to take into account the surface heat transfer conditions in the form of the equivalent heat sources.

The conductive lengths which limit the conductance influence along the rods are assumed to be much smaller than the corresponding geometrical lengths of the elements:

$$Lc_i = \left(\frac{k_i \delta_i}{h_i}\right)^{1/2} \ll L_i, \quad i = 1, \ldots, N. \tag{2}$$

This condition allows us to omit the longitudinal derivative in the conductivity equations for the rods and to neglect the conductive heat fluxes between the elements through the conjunctions, as shown in ref. [4].

Taking into account the above mentioned conditions, the temperature regimes of the elements can be described by the following ordinary differential equations:

$$\delta_i \rho_i c_i \frac{dT_i}{dt} = -h_i(t)(T_i - T_a(t)) + q_{si}(t),$$
$$i = 1, \ldots, N \tag{3}$$

where T_i are the temperatures of the truss elements, t is the time variable, ρ_i are the densities of the elements, c_i are the specific heats of the elements, $h_i(t)$, $T_a(t)$, $q_{si}(t)$ are the time-dependent functions of the convective heat transfer, ambient temperature and heat flux density, respectively.

For the case when the thermal time constants of all the elements are much smaller than the characteristic times of the heat transfer parameters variations:

$$t_i = \frac{\delta_i \rho_i h_i}{k_i} \ll \Delta t_h, \Delta t_T, \Delta t_q, \quad i = 1, \ldots, N \tag{4}$$

the temperature regimes of the elements can be considered as quasi-stationary. For this limit it is possible to neglect the time derivatives in equation (3). The temperatures of the elements for this limit can be given by the following expression:

$$T_i(t) = T_a(t) + \frac{q_{si}(t)}{h_i(t)}, \quad i = 1, \ldots, N \qquad (5)$$

which results from the energy balances on the surfaces of the elements. This expression shows that under quasi-stationary conditions (4) the heat transfer uniformity on the truss elements ($q_{si} = q_{sj}$; $h_i = h_j$; $i, j = 1, \ldots, N$) keeps the temperature uniformity in the truss construction and the temperature differences between the elements do not exist:

$$\Delta T_{ij} = T_i - T_j = 0.$$

But if the conditions (4) are not valid the dynamical temperature regimes which make up the subject of this paper take place.

3. DYNAMICAL HEAT EXCHANGE REGIMES

Taking into account that the parameters h_i, T_a, q_{si} are time-dependent one can obtain the solution of equation (3) in the general form [5] as:

$$T_i(\tau) = T_{i|\tau=0} \exp\left\{-\int_0^\tau h^*(\tau')\,\mathrm{d}\tau'\right\} + \int_0^\tau \{T_{si}^*(\tau') + h_i^*(\tau')T_a(\tau')\} \times \exp\left\{-\int_{\tau'}^{\tau} h_i^*(\tau'')\,\mathrm{d}\tau''\right\}\mathrm{d}\tau' \qquad (6)$$

where $\tau = t/t_i$ is the dimensionless time variable, $t_i = \delta_i \rho_i c_i / h_i(0)$ is the thermal time constant of the elements, $h_i^*(\tau) = h_i(\tau)/h_i(0)$ is the dimensionless convective heat transfer coefficient, $T_{si}^* = q_{si}(\tau)/h_i(0)$ is the temperature addition of the elements associated with the heat transfer on the elements surfaces and $T_{i|\tau=0}$ is the initial temperature of the elements.

This solution allows one to carry out the analysis of the dynamical thermal regimes and the temperature differences which originate from the time-dependent heat transfer operating parameters.

4. AMBIENT TEMPERATURE INFLUENCE

4.1. *Stepwise changes of the ambient temperature*

In this case it is assumed that the ambient temperature changes from T_{a1} to T_{a2} during the time Δt_T which is much smaller than the thermal time constants of the elements:

$$\Delta t_T \ll t_i, \quad i = 1, \ldots, N.$$

Taking into account the above conditions, and assuming that the initial temperature is given by the heat transfer balance approximation:

$$T_{i|\tau=0} = T_{a1} + \frac{q_s}{h} \qquad (7)$$

and completing the integration of the solution (6) one can obtain the following expression for the temperatures of elements:

$$T_i(t) = \frac{q_s}{h} + T_{a2} + (T_{a1} - T_{a2}) \exp(-t/t_i). \qquad (8)$$

This expression shows that the temperature of the elements changes exponentially and the rates of the change are determined by the thermal time constants which include the thermophysical properties, heat transfer coefficient and geometrical parameters. The temperature differences between two arbitrary elements are given by:

$$\Delta T_{ij} = (T_{a1} - T_{a2})(\exp(-t/t_i) - \exp(-t/t_j)). \qquad (9)$$

In accordance with this expression the temperature difference between the two elements is zero at the initial moment and grows up to its maximal value:

$$\max\{T_{ij}\} = (T_{a1} - T_{a2})(\eta_{ij}^{1/(1-\eta_{ij})} - \eta_{ij}^{\eta_{ij}/(1-\eta_{ij})}) \qquad (10)$$

where $\eta_{ij} = t_i/t_j$ is the ratio of the thermal time constants of the elements. This maximum is attained at the moment:

$$t_{\max} = (t_i^{-1} - t_j^{-1}) \ln \frac{t_j}{t_i}. \qquad (11)$$

After this moment the temperature difference decreases monotonously and for the times:

$$t > t_{\max} \ln \frac{|T_{a1} - T_{a2}|}{\delta T} \qquad (12)$$

it can be considered as negligible (δT is the temperature difference limit allowable between the elements of the truss).

In Fig. 1 the dimensionless temperature difference $\Delta T_{ij}/\Delta T_a$ vs the time variable are shown for the different values of the convective heat transfer coefficient h. The maximal value of $\Delta T_{ij}/\Delta T_a$ is determined by the following ratio:

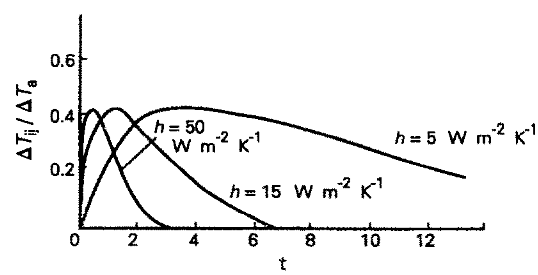

FIG. 1. Dimensionless temperature difference between elements vs time for the different convective heat transfer coefficients.

$$\eta_{ij} = \frac{\delta_i \rho_i c_i}{\delta_j \rho_j c_j}$$

and does not depend on the value of the coefficient h. The dying time of the temperature difference is proportional to the value of h^{-1} as can be seen from equations (11) and (12).

4.2. *Oscillation of the ambient temperature*

Now let us consider the case when the ambient temperature is given by the oscillatory function:

$$T_a(t) = \bar{T}_a + \Delta T_a \sin(\omega t) \quad (13)$$

where $\bar{T}_a$, ΔT_a are the harmonic mean and the amplitude of the ambient temperature, correspondingly.

Substituting this function in the general solution (6) and integrating it one can obtain the following time dependence for the temperature of the elements:

$$T_i(\tau) = \left(T_{i|\tau=0} - \frac{q_i}{h} - T_a + \frac{\bar{\omega}_i \Delta T_a}{1+\bar{\omega}_i^2}\right)\exp(-\tau)$$

$$+ \frac{q_s}{h} + \bar{T}_a + \frac{\Delta T_a}{1+\bar{\omega}_a^2}(\sin(\bar{\omega}_i \tau) - \bar{\omega}_i \cos(\bar{\omega}_i \tau)) \quad (14)$$

where $\bar{\omega}_i = \omega t_i = 2\pi t_i / t_p$ are the dimensionless frequencies which include the ratios of the thermal time constants to the period of the ambient temperature oscillations t_p.

This solution includes two components: the exponentially decreasing component dependent on the initial value and the periodic component corresponding to the response of the element on the ambient temperature oscillations. The first one vanishes for the times $t > (2 \div 3) t_i$. Let us consider the periodic component:

$$T_i(\tau) = \frac{q_i}{h} + \bar{T}_a + \frac{\Delta T_a}{1+\bar{\omega}_i^2}(\sin(\bar{\omega}_i \tau) - \bar{\omega}_i \cos(\bar{\omega}_i \tau)). \quad (15)$$

This expression shows that the amplitude and the phase of the temperature oscillation of the element depend on the dimensionless frequency $\bar{\omega}_i$. In the limit $\bar{\omega}_i \ll 1$ the amplitude and the phase of the temperature oscillation of the element do not differ noticeably from those of the ambient temperature and can be approximated by (13). In the limit $\bar{\omega}_i \gg 1$ the oscillatory component is negligible and the temperature of the element can be approximated by the following steady state value:

$$T_i \approx \bar{T}_a + \frac{q_s}{h}.$$

So two kinds of the quasi-stationarity are shown to exist. Within the limit $\bar{\omega}_i \ll 1$ the first kind of the quasi-stationary regime is established for which the temperature of the element follows simultaneously the ambient temperature oscillation without noticeable delay. In the limit $\bar{\omega}_i \gg 1$ the other kind of the quasi-stationarity takes place for which the temperature of the element does not respond to the ambient temperature oscillation and keeps on to be constant.

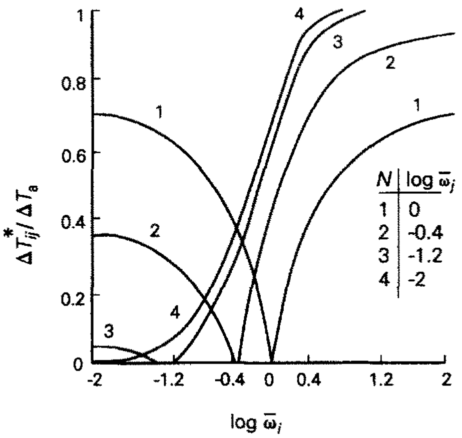

FIG. 2. Dimensionless amplitude of the temperature difference between the elements depending on the corresponding dimensionless frequencies.

The temperature difference between two elements with different values of the dimensional frequency is given by:

$$\Delta T_{ij} = \frac{\Delta T_a}{(1+\bar{\omega}_i^2)(1+\bar{\omega}_j^2)}\{(\bar{\omega}_j^2 - \bar{\omega}_i^2)\sin(\omega t)$$

$$+ (\bar{\omega}_j(1+\bar{\omega}_i^2) - (\bar{\omega}_i(1+\bar{\omega}_j^2))\cos(\omega t)\}. \quad (16)$$

This expression shows that the temperature differences between the elements are the periodic functions with their amplitude and phase dependent on the dimensionless frequencies values $\bar{\omega}_i$ and $\bar{\omega}_j$. The amplitude of the function (16) can be taken as the characteristic evaluation of the temperature difference between two elements. This value is given by:

$$\Delta T^*_{ij} = \frac{\Delta T_a}{(1+\bar{\omega}_i^2)(1+\bar{\omega}_j^2)}\{(\bar{\omega}_j^2 - \bar{\omega}_i^2)^2$$

$$+ (\bar{\omega}_i(1+\bar{\omega}_j^2) - \bar{\omega}_j(1+\bar{\omega}_i^2))^2\}^{1/2}. \quad (17)$$

The amplitude of the temperature difference is proportional to the ambient temperature amplitude ΔT_a and is dependent on the oscillation period and on the thermal time constants of the considered pair of elements. The dimensionless temperature difference amplitude $\Delta T^*_{ij}/\Delta T_a$ is shown in Fig. 2 as a function of the values $\bar{\omega}_i$ and $\bar{\omega}_j$. The temperature difference vanishes for the case when $\bar{\omega}_i = \bar{\omega}_j$. It is evident that in the system of the many elements the most significant value of $\Delta T^*_{ij}/\Delta T_a$ corresponds to the pair of the elements with the maximal and minimal thermal time constants.

5. SURFACE HEAT FLUX INFLUENCE

5.1. *Stepwise change of the heat flux*

In this case it is assumed that the surface heat flux density changes from q_{s1} to q_{s2} during the time Δt_q

which is much smaller than the thermal time constants of the elements:

$$\Delta t_q \ll t_i, \quad i = 1, \ldots, N.$$

Taking into account the conditions mentioned, and assuming that the initial temperature is given by the heat transfer balance approximation:

$$T_{i|t=0} = T_a + \frac{q_s}{h} \tag{18}$$

and completing the integration of the general solution (6) one can obtain the following time dependence for the temperature of the elements:

$$T_i(t) = T_a + \frac{q_{s2}}{h} + \frac{q_{s1} - q_{s2}}{h} \exp(-t/t_i). \tag{19}$$

This expression is similar to (8). The temperature difference between the two arbitrary elements is given by:

$$\Delta T_{ij} = \frac{q_{s1} - q_{s2}}{h} (\exp(-t/t_i) - \exp(-t/t_j)). \tag{20}$$

The maximal value of the temperature difference:

$$\max \{\Delta T_{ij}\} = \frac{q_{s1} - q_{s2}}{h} (\eta_{ij}^{1/(1-\eta_{ij})} - \eta_{ij}^{\eta_{ij}/(1-\eta_{ij})}) \tag{21}$$

is attained at the moment:

$$t_{max} = (t_i^{-1} - t_j^{-1}) \ln \frac{t_j}{t_i}. \tag{22}$$

For the times:

$$t > t_{max} \ln \frac{|q_{s1} - q_{s2}|}{h\delta T} \tag{23}$$

this difference can be neglected.

The expressions (19)–(23) are similar to those in the case 4.1. Some distinctions should be pointed out. Firstly, the maximum temperature difference for this case is in inverse proportion to the convective heat transfer coefficient. Secondly, the dying time of the temperature difference is more strongly dependent on the value of h in comparison with that of 4.1.

5.2. *Oscillation of the heat flux*

Let us consider the case, when the surface heat flux density is given by the oscillatory function:

$$q_s(t) = \bar{q}_s + \Delta q_s \sin(\omega t) \tag{24}$$

where $\bar{q}_s$, Δq_s are the harmonic mean and the amplitude of the surface heat flux density, correspondingly.

Substituting this function in the general solution (6) and integrating it one can obtain the following time dependence for the temperature of the elements:

$$T_i(\tau) = \left(T_{i|\tau=0} - \frac{\bar{q}_s}{h} - T_a + \frac{\Delta q_s \bar{\omega}_i}{h(1+\bar{\omega}_i^2)} \right) \exp(-\tau)$$

$$+ \frac{\bar{q}_s}{h} + T_a + \frac{\Delta q_s}{h(1+\bar{\omega}_i^2)} (\sin(\bar{\omega}_i \tau) - \bar{\omega}_i \cos(\bar{\omega}_i \tau)). \tag{25}$$

Considering only the periodic component of this solution:

$$T_i(\tau) = \frac{\bar{q}_s}{h} + T_a + \frac{\Delta q_s}{h(1+\bar{\omega}_i^2)} (\sin(\bar{\omega}_i \tau) - \bar{\omega}_i \cos(\bar{\omega}_i \tau)) \tag{26}$$

one can find the periodic temperature difference between the two arbitrary elements as:

$$\Delta T_{ij} \frac{\Delta q_s}{h(1+\bar{\omega}_i^2)(1+\bar{\omega}_j^2)} \{(\bar{\omega}_j^2 - \bar{\omega}_i^2) \sin(\omega t)$$

$$+ (\bar{\omega}_j(1+\bar{\omega}_i^2) - \bar{\omega}_i(1+\bar{\omega}_j^2)) \cos(\omega t)\} \tag{27}$$

and the amplitude as the characteristic evaluation of the temperature difference between the elements:

$$\Delta T_{ij}^* = \frac{\Delta q_s}{h(1+\bar{\omega}_i^2)(1+\bar{\omega}_j^2)} \{(\bar{\omega}_j^2 - \bar{\omega}_i^2)^2$$

$$+ (\bar{\omega}_i(1+\bar{\omega}_j^2) - \bar{\omega}_j(1+\bar{\omega}_i^2))^2\}^{1/2}. \tag{28}$$

The evaluations, obtained in this section, are similar to those of the case 4.2. The dependences for the dimensionless complexes $h\Delta T_{ij}/\Delta q_s$ and $h\Delta T_{ij}^*/\Delta q_s$ are the same as for $\Delta T_{ij}/\Delta T_a$ and $\Delta T_{ij}^*/\Delta T_a$.

6. CONVECTIVE HEAT TRANSFER INFLUENCE

6.1. *Stepwise change of the convective heat transfer*

Let us consider the case when the convective heat transfer coefficient changes from h_1 to h_2 during the time which is much smaller than the thermal time constants of the elements:

$$\Delta t_h \ll t_i, \quad i = 1, \ldots, N.$$

Taking into account the above mentioned conditions, assuming that the initial temperature is given by the heat fluxes balance approximation:

$$T_{i|t=0} = T_a + \frac{q_s}{h_1} \tag{29}$$

and integrating the general solution (6) one can obtain the following time dependence for the temperature of the elements:

$$T_i(t) = T_a + \frac{q_s}{h_2} + \frac{(h_2 - h_1) q_s}{h_2 h_1} \exp(-t/t_i) \tag{30}$$

where $t_i = \delta_i \rho_i c_i / h_2$ are the thermal time constants corresponding to the latest value of the convective heat transfer coefficient.

This expression allows one to obtain the temperature difference between the two arbitrary elements:

$$\Delta T_{ij} = \frac{(h_2 - h_1) q_s}{h_1 h_2} (\exp(-t/t_i) - \exp(-t/t_j)) \tag{31}$$

and its maximal value:

$$\max\{\Delta T_{ij}\} = \frac{(h_2-h_1)q_s}{h_1h_2}\left(\eta_{ij}^{1/(1-\eta_{ij})}-\eta_{ij}^{\eta_{ij}/(1-\eta_{ij})}\right) \tag{32}$$

which is attained at the moment:

$$t_{max} = (t_i^{-1}-t_j^{-1})\ln\frac{t_j}{t_i}. \tag{33}$$

All these expressions are fully similar to the evaluations of Sections 4.1 and 5.1.

6.2. *Oscillatory convective heat transfer*

Let us consider the case when the convective heat transfer coefficient is given by the oscillatory function:

$$h(t) = \bar{h}+\Delta h\sin(\omega t) \tag{34}$$

where $\bar{h}$, Δh are the harmonic mean and the amplitude of the convective heat transfer coefficient.

Substitution of this function in the general solution (6) gives the following expression for the temperature of the elements:

$$T_i(\tau) = (T_{i|\tau=0}-T_a)\exp\left(-\tau-\frac{\Delta h^*}{\bar{\omega}_i}(1-\cos(\bar{\omega}_i\tau))\right)$$

$$+T_a+\frac{q_s}{\bar{h}}\int_0^{\tau}\exp(\tau'-\tau)$$

$$\times\exp\left(\frac{\Delta h^*}{\bar{\omega}_i}(\cos(\bar{\omega}_i\tau)-\cos(\bar{\omega}_i\tau'))\right)\mathrm{d}\tau' \tag{35}$$

where $\Delta h^* = \Delta h/\bar{h}$ is the dimensionless amplitude of the oscillatory heat transfer coefficient.

The solution (35) contains the exponentially decreasing component and the periodic component. In Fig. 3(a) the time dependences of the temperature of the elements with the following values of the dimensionless frequency $\bar{\omega}_i = 10^{-1}$ and $\bar{\omega}_i = 10$ are shown. The initial temperature of the elements is assumed to be $T_{i|\tau=0} = T_a$. These graphs describe the time dependence of the dimensionless temperature $(T_i(\tau)-T_a)\bar{h}/q_s$. The calculations were made by means of the numerical integration procedure [6].

The graphs in Fig. 3(a) show the fast transition to the periodic temperature regime in case of the small value of $\bar{\omega}_i$ and the slow transition in case of the high value of $\bar{\omega}_i$. In Fig. 3(b) the stabilized periodic regimes are shown for the different dimensionless frequencies $\bar{\omega}_i$ of the oscillations of the convective heat transfer coefficient. In the limit $\bar{\omega}_i \to 0$ the temperature of the element follows the time-dependent heat transfer without the noticeable delay and can be described by the surface heat transfer balance approximation given by the expression:

$$T_i(t) = T_a+\frac{q_s}{\bar{h}+\Delta h\sin(\omega t)}. \tag{36}$$

In the limit $\bar{\omega}\to\infty$ the temperature of the element does not respond to the oscillation of the convective heat transfer coefficient and can be approximated by the following evaluation:

$$T_i(t) = T_a+\frac{q_i}{\bar{h}}. \tag{37}$$

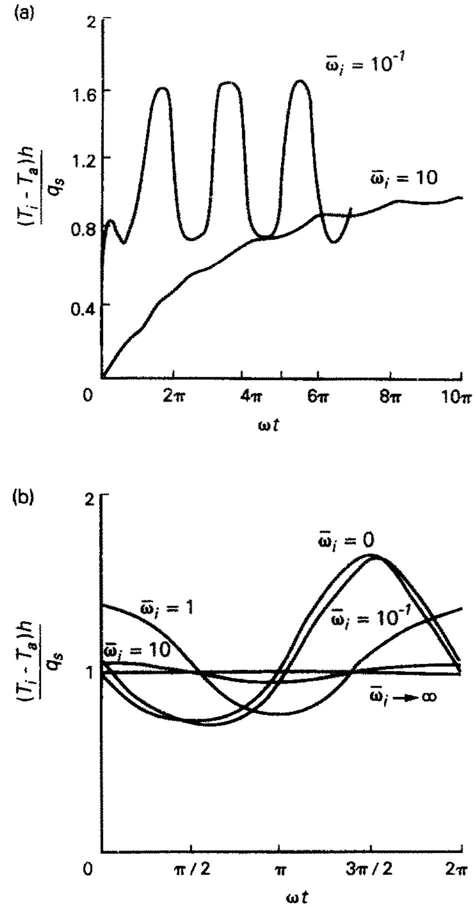

FIG. 3. (a) Stabilization of the periodic temperatures of the elements under oscillatory convective heat transfer. (b) Periodic temperatures of the elements under oscillatory convective heat transfer for the different values of the dimensionless frequency.

In Fig. 4 the dependence of the dimensionless amplitude of the temperature difference $\Delta T_{ij}^*\bar{h}/q_s$ between the two arbitrary elements is presented for the different values of $\bar{\omega}_i$ and $\bar{\omega}_j$. The values of $\Delta T_{ij}^*\bar{h}/q_s$ were obtained by means of the numerical simulation program which integrates the expression (35) for the two values $\bar{\omega}_i$, $\bar{\omega}_j$ and finds the maximum of the temperature difference ΔT_{ij} in the stabilized periodic regime between the considered pair of the elements. These graphs are in qualitative similarity with those shown in Fig. 2 for the case of the oscillation of the ambient temperature. Comparing the cases it should be noted only that the values of $\Delta T_{ij}^*\bar{h}/q_s$ are dependent in addition on the dimensionless amplitude of the heat transfer coefficient Δh^*.

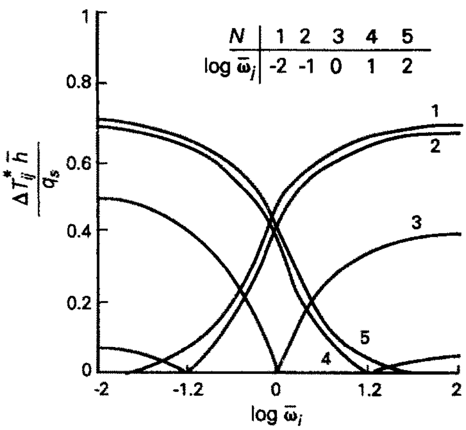

FIG. 4. Dimensionless amplitude of the temperature difference between the elements depending on the corresponding dimensionless frequencies.

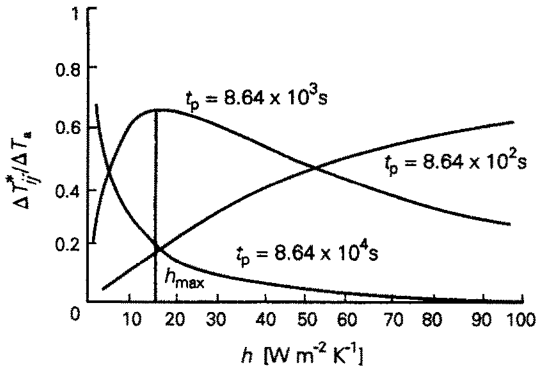

FIG. 5. Dimensionless amplitude of the temperature difference between the elements vs convective heat transfer coefficient for the ambient air temperature oscillation.

7. TEMPERATURE DIFFERENCES SUPPRESSION

In the above sections of the paper we analysed the dynamical mechanisms of the temperature differences between the elements of the truss constructions associated with stepwise changing and oscillatory heat transfer conditions. The understanding of the possible ways of the suppression of these temperature differences is of practical interest, especially, for the thermal control of the truss constructions of precise radio telescopes [1, 3].

The formulas (17), (28), (35), derived for the cases of the oscillatory parameters demonstrate that the one possible way is associated with the decrease of the oscillation amplitudes of the heat transfer parameters: ΔT_a, Δq_s, Δh. This mean allows one to diminish the amplitude of the temperature oscillations of the elements and, thereby, the amplitudes of the temperature differences between them. For the oscillations of T_a and q_s the amplitude of the temperature difference between the elements is proportional to the corresponding amplitudes ΔT_a and Δq_s. But this mean is applicable only in the case if the control of the amplitudes of the operating heat transfer parameters is feasible.

Another possible way of controlling the dynamical temperature differences consists of intensifying the convective heat transfer on the surface of the truss by means of the forced ventilation of the ambient air through it. In Fig. 5 the amplitude of the dimensionless temperature difference between two elements is presented as a function of the convective heat transfer coefficient for the case of ambient temperature oscillations. The calculations are made for the values of $\rho c\delta_i = 10^4$ J m^{-2} K^{-1} and $\rho c\delta_j = 5\times10^4$ J m^{-2} K^{-1} for the different values of the frequency ω of the ambient air temperature. The possible intervals for the convective heat transfer coefficients were evaluated by means of the criterion formulas for the tubes in forced and buoyancy driven air flows [7]. Curve 1 is calculated for the value $\omega = 2\pi/3600\times24$ s^{-1} which corresponds to the daytime cycle of the ambient air temperature oscillation. For this case the value of $\Delta T^*_{ij}/\Delta T_a$ decreases proportionally to h^{-2}. The temperature difference decreasing is explained by the fact that the increase of h leads to the diminishing of the thermal time constants of the elements. The amplitude and the phase of the elements approach the air's parameters in this case.

The parametric study of the expression (17) shows an interesting effect. This effect consists in the temperature differences which increase along with the increase in the convective heat transfer coefficient. Curve 2 in Fig. 5 shows the temperature difference amplitude between two elements for the same values of $\rho c\delta$ but for the other value of the frequency $\omega = 2\pi/360\times24$ s^{-1}. For these operating conditions the temperature difference amplitude dependence on h has a maximum. This maximum is explained by the fact that for the low values of h the following relations $\bar{\omega}_i \gg 1$, $\bar{\omega}_j \gg 1$ are fulfilled for both elements. In these operating conditions the temperatures of the elements do not respond to the ambient air temperature oscillations. The increase in the coefficient h diminishes the values of $\bar{\omega}_i$ and $\bar{\omega}_j$ and the temperature oscillations of the elements approach the air temperature oscillation. But the point is that this approach occurs differently for the elements with the different values of $\delta\rho c$ which determine the thermal inertia of the elements. As a result the temperature difference grows up to its maximal value and only then goes down along with the increase of h. The further decrease is explained by the fact that for this interval of h the temperature regimes of the elements approach the air temperature oscillation and, thereby, approach each other, i.e. for these heat transfer operating conditions the relations $\bar{\omega}_i \ll 1$, $\bar{\omega}_j \ll 1$ are valid.

Let us discuss the influence of the heat transfer coefficient on the temperature differences, associated

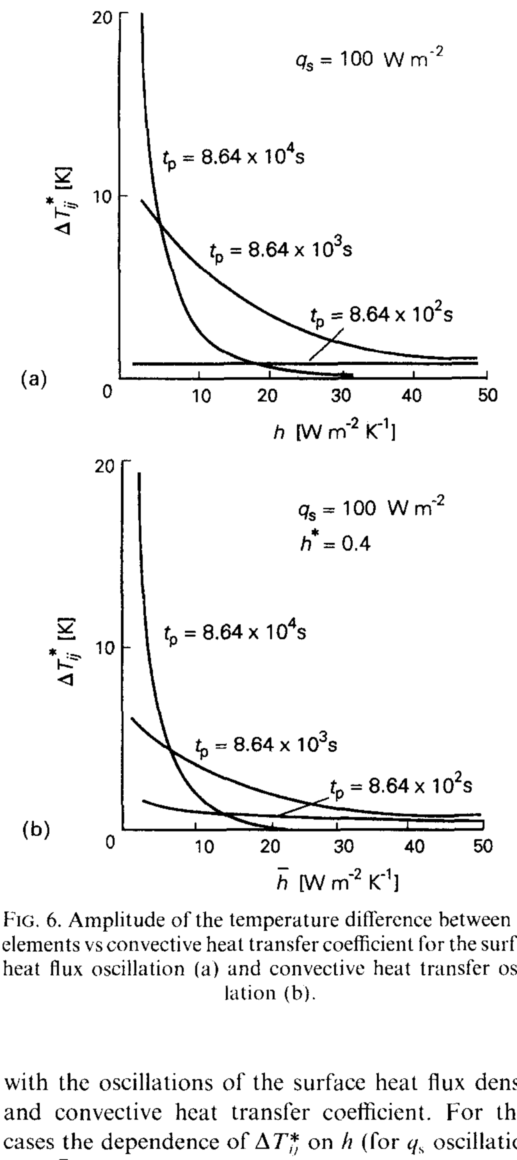

FIG. 6. Amplitude of the temperature difference between the elements vs convective heat transfer coefficient for the surface heat flux oscillation (a) and convective heat transfer oscillation (b).

with the oscillations of the surface heat flux density and convective heat transfer coefficient. For these cases the dependence of ΔT^*_{ij} on h (for q_s oscillation) and $\bar{h}$ (for h oscillation) are significantly stronger, because the values of h or $\bar{h}$ are included not only in the complexes $\bar{\omega}_i$ and $\bar{\omega}_j$ as it takes place in case of the ambient temperature oscillations. In accordance with the formulas (28) and (35) the inverse proportions between the values ΔT^*_{ij} and h take place as well. Figure 6(a) presents the dependence of ΔT^*_{ij} on h for the values $\rho c\delta_i$, $\rho c\delta_j$ given above and ω in the case of the oscillation of the heat flux density on the surface of the elements. Figure 6(b) presents the similar dependence in the case of the oscillation of the convective heat transfer. Both cases show the monotonous decrease of the temperature difference amplitude between the elements along with the increase of the convective heat transfer coefficient within the considered interval of h.

8. CONCLUSIONS AND SUMMARY

Analysis of the dynamical mechanism of temperature difference arising between the elements of truss constructions under time-dependent uniform heat transfer conditions is performed. This analysis considers the influence of the following parameters: the ambient temperature; the surface heat flux density; and the convective heat transfer.

In the case of stepwise changes of the mentioned parameters the corresponding temperature differences between the elements increase until they reach the maximal value, then decrease monotonously and vanish exponentially. The dying time of the temperature differences is in inverse proportion to the convective heat transfer coefficient that allows suppression of them by means of the convective heat transfer intensification on the truss surface.

In the case of the oscillatory heat transfer conditions the elements of the truss constructions have the periodic temperatures, the amplitude and the phase of which depend on the thermal time constants of the elements. The different thermal time constants of the elements lead to the oscillatory temperature differences between them under uniform time-dependent operating conditions. The temperature differences are shown to be small for the two operating conditions limits. Firstly, in the case, when the thermal time constants of the elements are much larger than the period of the oscillation of the heat transfer conditions. Secondly, in the case, when the thermal time constants of the elements are much smaller than the period of heat transfer conditions oscillations. For the first limit of the operating conditions the temperature regimes of the elements respond weakly on the heat transfer parameters oscillations and the corresponding temperature differences are very small and can be neglected. For the second limit of the operating conditions the temperature regimes of the elements follow the oscillations of the heat transfer parameters simultaneously and the corresponding temperature differences between them are very small and can also be neglected.

Under the oscillatory ambient temperature the convective heat transfer coefficient increase leads to the decrease of the temperature differences between the elements, only for operating conditions for which the dimensionless frequencies of the element temperatures oscillations are smaller than unity. In the case when these relations are not fulfilled the increase of the heat transfer coefficient leads to the increase of the temperature differences between the elements due to the fact that the temperature regimes of the elements approach the oscillatory ambient temperature differently. Only after attaining the maximum level do the temperature differences between the elements begin to decrease along with the further increase of the heat transfer coefficient. And in the case of these operating conditions the more effective way of suppression, of the temperature differences between the elements of the truss constructions, consists in the increase of the thermal resistance between the elements surfaces and the ambient airflow, for example, by means of the thermal insulation.

For the case of the oscillations of the heat flux and convective heat transfer on the surface of the truss elements increase in the heat transfer coefficient leads to the monotonous decreasing of the corresponding temperature differences within the considered interval of operating parameters.

It should be pointed out that the obtained analytical evaluations allow one to perform calculations of the temperature differences between the elements of truss constructions under uniform time-dependent heat transfer conditions and to analyse the influence of the different operating and structural parameters. These evaluations allow also to determine the parameters of the convective heat transfer regime necessary to provide the radio telescopes with the reliable means of the thermal control of supporting truss constructions.

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LES MECANISMES DYNAMIQUES DE DIFFERENCES DE TEMPERATURE, SURGISSANTS EN CONDITIONS HOMOGENES DE L'ECHANGE THERMIQUE

Résumé—L'article est dévouée à l'étude analytique de différences de température entre des éléments constructifs des fermes en conditions homogènes de l'échange thermique. Les différences de température surgissant comme le resultat des différences entre des constantes thermiques du temps des éléments. Les éléments de ferme sont considérés comme les corps thermiquement minces. Les longueurs conductives sont supposées beaucoup moins que les dimensions longitudinales des élément, ce que permet de négliger le transfer conductif de chaleur. Les évaluations des différences thermiques sont faites pour les suivants paramètres, lesquells se changent brusquement et d'une manière oscillatoire : la température de courant d'aire, le flux de chaleur sur la surface de ferme et l'intensité de l'échange thermique sur la surface des éléments. Les différences de température sont négligeables pour deux cas limits de conditions du fonctionnement. Premièrement, pour les régimes, quand toutes les constantes thermiques de temps sont beaucoup plus grandes que la période du changement de paramètres opérationnels. Secondement, pour les régimes, quand toutes les constantes thermiques du temps sont beaucoup moins que la période du changement de paramètres opérationnels.

DIE DYNAMISCHEN MECHANISMUSSE DER ENTSTEHUNG DER TEMPERATURDIFFERENZ BEI GLEICHARTIGE WÄRMEAUSTAUSCHBEDIGUNGEN

Zusammenfassung—In diesem Artikel wird die Analyse der dynamischen Mechanismusse der Entstehung der Temperaturdifferenz zwischen Elemente der trägerenen Aufbau, wie in gleichartige Wärmeaustauschbedigungen sich befinden, durchgeführt. Die unstationären Temperaturdifferenz zwischen Elemente der Trägeren enstehen im Ergebris der Untershied siener Wärmekonstante der Zeit. In der Arbeit wird die Elemente der Trägeren wie die thermischfein Körper angesehen. Die konductive Länge der Elemente wird für vielweniger der Geometrischlänge gegolten, daß die konductiven Wärmeübergabe verschmähen wird zugelassen. In der Artikel wurden die Schätzungen der entstehenen Temperaturdifferenz bei sprunghaften und periodischen Anderungen folgende Wärmeaustauschbedigungen durchgeführt : der Luftstrometemperatur, der Wärmeentwicklungintensität in Elementeoberfläche, der Konvectionenwärmeaustauschintensität. Man zeigt, daß die Temperaturdifferenz in zwei Grenzfälle sind klein : erstens, bei den Regime, wenn die Zeitkonstante alle Elemente viel größer als der Periode der Änderung der Wärmeaustauschparameter sind ; zweitens, bei den Regime, wenn die Zeitkonstante alle Elemente viel kleiner als der Periode der Änderung der Wärmeaustauschparameter sind.

ДИНАМИЧЕСКИЕ МЕХАНИЗМЫ ВОЗНИКНОВЕНИЯ ТЕМПЕРАТУЯНЫХ ПЕРЕПАДОВ ПРИ ОДНОРОДНЫХ УСЛОВИЯХ ТЕПЛООБМЕНА

Аннотация—Аналитически исследуются динамические механизмы возникновения разностей температур между элементами ферменных конструкций при однородных условиях теплообмена. В случае стационарных однородных условий разности температур отсутсвуют, в то время как динамические нестационарные параметры теплообмена приводят к возникновению разностей температур даже при однородных условиях. Указанные нестациоиарные разности температур связаны с различиями тепловых постоянных времени элементов. Аналитически оцениваются разности температур для скачкообразно изменяющихся и колебательных параметров теплообмена : температуры воздушного потока, интенсивнозти тепловыделения на поверхности элементов и коэффициентов конвективного теплобмена. Показано, что для предельных случаев указанных рабочих условий разности температур невелики. Предельными являются режимы, когда, во-первых, все постоянные времени элементов существенно превосходят период колебательных параметров теплообмена и, во-вторых, когда постоянные времени всех элементов значительно меньше колебательного периода.